Reynolds number: a) A laminar separation bubble of limited spanwise extent first appears in the middle of the wing, with attached laminar flow on either side. b) Onset of transition in the wing boundary layer causes a local collapse of separation in the middle, while laminar separation persists outboard, producing a characteristic "two-lobed" separation pattern. c) The outboard lobes of laminar separation spread to the extremities of the hinge line, the inner (transitional or turbulent) flow remaining attached. The three examples illustrated in Fig. 4 have been observed in flow visualization studies reported in Ref. 1.

Conclusion

Trailing-edge flap-induced separation data on 75° delta wings have been examined to determine incipient separation characteristics. While the delta wing data for the turbulent boundary layer correlate with two-dimensional results, in the laminar and transitional cases a nearly parallel shift to higher flap angles for incipient separation is found. More data are needed (particularly for a range of leading-edge sweep angle) to establish properly the distinctive features of incipient separation on delta wings highlighted in this Note.

References

[1] Rao, D. M., "Hypersonic Control Effectiveness Studies on Delta Wings with Trailing-Edge Flaps," Ph. D. Thesis, Aeronautics Dept., Imperial College, London University, 1970.

[2] Davies, L., private communication (1970).

[3] Keyes, J. W., "Pressure and Heat Transfer on a 75° SweptDelta Wing with Trailing-Edge Flap at Mach 6 and Angle of Attack to 90°," NASA TN D-5418, Sept. 1969.

[4] Whitehead, A. H., Jr., and Keyes, J. W., "Flow Phenomena and Separation over Highly Swept Delta Wings with Trailing-Edge Flaps at Mach 6," *AIAA Journal*, Vol. 6, Dec. 1968, pp. 2380-2387.

[5] Johnson, C.B., "Pressure and Flowfield Study at Mach Number 8 of Flow Separation on a Flat Plate with Deflected Trailing-Edge Flap," NASA TN D-4308, March 1968.

[6] Needham, D. A. and Stollery, J. L., "Boundary-Layer Separation in Hypersonic Flow," AIAA Paper 66-455, Los Angeles, Calif., 1966.

[7] Rao, D. M., "An Experimental Study of the Hypersonic Aerodynamics of Delta Wings," *Journal of the Aeronautical Society of India*, Vol. 23, Nov. 1971, pp. 183-190.

[8] Stollery, J. L., "Laminar and Turbulent Boundary-Layer Studies at Hypersonic Speeds," ICAS Paper 72-09, International Council of Aeronautical Sciences, London, 1972.

[9] Putnam, L. E., "Investigation of Effects of Ramp Span and Deflection Angle on Laminar Boundary-Layer Separation at Mach 10.03," NASA TN D-2833, May 1965.

Postbuckling Analysis of Crossply Laminated Plates

Ramesh Chandra*

National Aeronautical Laboratory, Bangalore, India.

Nomenclature

a	= plate length in x direction
b	= plate width in y direction
w	= deflection of a point on median surface of plate in direction normal to the undeformed plate
E_L, E_T	= Young's moduli of lamina in parallel to fibre and perpendicular to fiber directions, respectively
G_{LT}	= shear modulus of lamina in LT plane
μ_{TL}, μ_{LT}	= Poisson's ratios; first subscript denotes lateral direction, second denotes load direction
E_f, E_m	= Young's moduli of fibre and matrix, respectively
m	= crossply ratio (ratio of the total thickness of odd layers to the total thickness of even layers)
n	= total number of layers
r	= a/b, aspect ratio of the plate
V_m	= volume content of matrix
P	= applied load
P_o	= buckling load
E	= E_L
F	= E_T/E_L
μ	= μ_{LT}
η	= G_{LT}/E_L
δ	= $\eta(F-\mu^2)/F$
C_{11}	= $EF/(\mathrm{N}-\mu^2)$
A	= amplitude of buckling mode

Received January 27, 1975; revision received April 1, 1975.

Index categories: Structural Composite Materials; Structural Stability Analysis.

*Scientist, Structural Sciences Division.

Introduction

POSTBUCKLING analysis of orthotropic plates is presented in Ref. 1. Chan[2] presented postbuckling analysis for arbitrarily laminated plates using energy methods. In this Note, an approximate solution (one term solution) to postbuckling problem of unsymmetrically stacked crossply laminated plate simply supported all over the edges is presented. The analysis is based on governing equations derived in Ref. 3.

Analysis

The governing equations for crossply laminated plates are obtained by deleting the inertia term from the governing equations of Ref. 3. These are recorded as follows:

$$L_1 w - L_3\phi - L_n(\phi, w) = 0 \tag{1a}$$

$$L_2\phi + L_3 w - \tfrac{1}{2}L_n(w, w) = 0 \tag{1b}$$

where ϕ is the Airy Stress Function.

$$L_1 = D^*_{11}(\partial^4/\partial x^4) + 2(D^*_{12} + 2D^*_{66})\ (\partial^4/\partial x^2\partial y^2) + D^*_{22}(\partial^4/\partial y^4)$$

$$L_2 = A^*_{22}(\partial^4/\partial x^4) + (2A^*_{12} + A^*_{66})\ (\partial^4/\partial x^2\partial y^2) + A^*_{11}(\partial^4/\partial y^4)$$

$$-L_3 = B^*_{21}(\partial^4/\partial x^4) + (B^*_{11} + B^*_{22})\ (\partial^4/\partial x^2\partial y^2) + B^*_{12}(\partial^4/\partial y^4)$$

$$L_n(\phi, w) = \phi,_{xx} w,_{yy} + \phi,_{yy} w,_{xx} - 2\phi,_{xy} w,_{xy}$$

A^*, B^*, and D^* are the matrices as defined in Ref. 4.

The plate is assumed to be simply supported at all the edges. The deflection function satisfying the geometric boundary conditions is assumed as follows:

$$w = A \sin(\pi x/a) \sin(\pi y/b) \tag{2}$$

It is not possible to satisfy the force boundary conditions by the previous function as the moment depends, not only upon the curvatures, but in-plane forces too, for unbalanced laminated plates. Hence, a modified galerkin's method[5] wherein the residues at the edges are minimized, is applied here.

Inplane boundary conditions considered are as follows:

$$\int_o^b (\phi,_{yy})_{x=o,a} dy = -P; \quad \int_o^b (\phi,_{xy})_{x=o,a} dy = o$$

$$\int_o^a (\phi,_{xx})_{y=o,b} dx = o; \quad \int_o^a (\phi,_{xy})_{y=o,b} dx = o \tag{3}$$

Substituting expression (2) in Eq. (1b), and solving it, ϕ satisfying Eq. (3) is obtained as,

$$\phi = \frac{N}{D} h A \sin\frac{\pi x}{a} \sin\frac{\pi y}{b} + \frac{A^2 h^2}{2} r^2 \left[\frac{1}{A^*_{22}} \cos\frac{2\pi x}{a} + \frac{1}{r^4 A^*_{11}} \cos\frac{2\pi y}{b} \right] - \frac{P}{2b} y^2 \tag{4}$$

where

$$N = B^*_{21} \frac{\pi^4}{a^4} + (B^*_{11} + B^*_{22}) \frac{\pi^4}{a^2 b^2} + B^*_{12} \frac{\pi^4}{b^4}$$

$$D = A^*_{22} \frac{\pi^4}{a^4} + (2A^*_{12} + A^*_{66}) \frac{\pi^4}{a^2 b^2} + A^*_{11} \frac{\pi^4}{b^4}$$

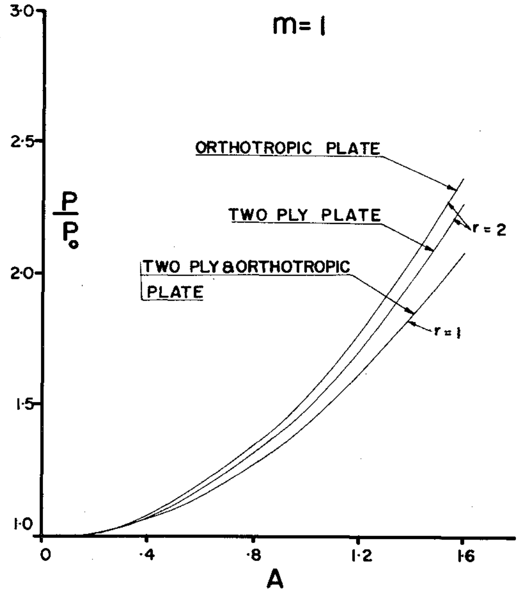

Fig. 1 Load-deflection curve for GFRP plate.

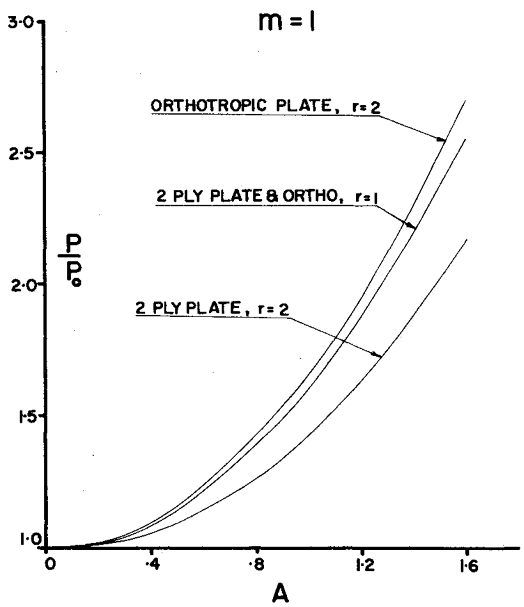

Fig. 2 Load-deflection curve for CFRP plate.

Substituting ϕ and w in Eq. (1a) and applying a modified Galerkin's techique, following algebraic equation in A is obtained.

$$(\alpha - \pi^2 (h/4a) P) A + \beta A^2 + \gamma A^3 = 0 \tag{5}$$

where

$$\alpha = (C_{11} h^4 / a^3) b \pi^4 [d^*_{11} + 2r^2 (d^*_{12} + \delta/6) + d^*_{22} r^4 + \{ b^*_{12} (r^4 - 1) + (b^*_{22} - b^*_{11}) r^2 \}^2 \times \{ 1/a^*_{22} + r^2 (2/a^*_{12} + 1/\delta) + r^4/a^*_{11} \}^{-1}] \tag{6}$$

$$\beta = 8/3 (C_{11} h^4 / a^3) b \pi^2 r^2 [b^*_{12} (a^*_{22} - a^*_{11}) - 4 \{ b^*_{12} (r^4 - 1) + (b^*_{22} - b^*_{11}) r^2 \} \times \{ 1/a^*_{22} + r^2 (2/a^*_{12} + 1/\delta) + r^4/a^*_{11} \}^{-1}] \tag{7}$$

$$\gamma = (C_{11} h^4 / 16 a^3) b \pi^4 (a^*_{11} + r^4 a^*_{22}) \tag{8}$$

The other parameters are as defined in Ref. 3.

Buckling load is determined from Eq. (5) by deleting A^2 and A^3 terms from it. Hence

$$P_o = (4a/h\pi^2)\ \alpha \tag{9}$$

Knowing applied load P in the postbuckling range, Eq. (5) can be easily solved for A. Thus the unknown amplitude in the linear buckling is determined. Knowing the deflection, stresses can easily be computed. Thus, the state of stresses and displacement in the postbuckling range is known.

Conclusion

Load deflection curves are plotted using Eq. (5) for GFRP (Glass Fibre Reinforced Plastics) and CFRP (Carbon Fibre Reinforced Plastics) plates. Material constants of these are taken from Ref. 3. It is noted from Figs. 1 and 2 that for a square plate (GFRP or CFRP) the bending-inplane coupling does not affect the load deflection curve much. But for rectangular plates this coupling influences the load-deflection curve quite a bit. It is also noted that this effect is more for CFRP plates as compared to GFRP plates. This is easily explained since for the 2 ply configuration, the bending-inplane coupling is more pronounced for CFRP plates as compared to GFRP plates. Figures 1 and 2 also reveal that for the same load less deflection is predicted by orthotropic solutions as compared to the 2 ply solutions. Here orthotropic solutions are obtained by neglecting bending-implane coupling. Thus, it is inferred that considering unbalanced laminates as orthotropic for this particular problem would predict nonconservative deflections.

References

[1]Chandra R. and Raju, B. B., "Postbuckling Analysis of Rectangular Orthotropic Plates,"*International Journal of Mechanical Sciences,* Vol. 15, Jan. 1973, pp. 81-97.

[2]Chan, D. P., "An Analytical Study of the Postbuckling of Laminated, Anisotropic Plates," Ph.D. thesis, Engineering Mechanics Dept., Case Western Reserve University, Cleveland, Ohio, 1970.

[3]Chandra, R. and Raju, B. B., "Large Amplitude Flexural Vibrations of Crossply Laminated Composite Plates," *Fibre Science and Technology,* to be published.

[4]Tsai, S. W., "Structural Behavior of Composite Materials," CR-71, 1964, NASA.

[5]Houbolt, I. C. and Brooks, G. W., "Differential Equations of Motion for Combined Flap-wise Bending, Chord-wise Bending and Torsion of Twisted Nonuniform Rotor Blades," NACA TN 3905, Feb. 1957, pp. 18.